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Second Law Analysis for Free Convection in an L-Shaped Cavity Filled with Nanofluid

Afroza Nahar, Salma Parvin, and M. Hasanuzzaman

***Abstract*—Natural convection heat transfer occurs in engineering applications like solar thermal collectors, electronic device cooling, nuclear reactors, etc. This paper aims to analyze the heat transfer and entropy generation in free convection laminar flow of nanofluid flowing through an L-shaped cavity using different nanoparticles. The second law of thermodynamics has been applied to investigate the effect of Prandtl number on the average Nusselt number, total entropy generation and Bejan number using water and, Cu-water, Ag-water and Al_2O_3-water nanofluids. Isotherms, stream function and entropy generation caused by heat transfer are also presented as a function of Prandtl numbers for various nanoparticles. Using the penalty finite element method with Galerkin's weighted residual, the governing equations are solved. Results show that Ag-water nanofluid with the highest Prandtl number gives the highest amount of irreversibility as well as rate of heat transfer. Cu-water and Ag-water nanofluid produce more irreversibilities than Al_2O_3-water nanofluid and base fluid. Also, Nusselt number and Bejan number increase with the increasing Prandtl number. Therefore, Prandtl number is a central parameter for desired heat transfer increment with decreasing entropy generation in the given geometry.**

***Index Terms*—penalty finite element method, free convection, L-shaped cavity, nanofluid, second law.**

Nomenclature:

Be	Bejan number
C_p	Specific heat at constant pressure ($J\,kg^{-1}\,K^{-1}$)
h	Heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
k	Thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	Length of the exterior surface (m)
L_1	Length of the interior surface (m)
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
T	Absolute temperature (K)
u, v	Dimensional velocity along x- and y-axis ($m\,s^{-1}$)
U, V	Dimensionless velocities
W	Width of the cavity (m)
x, y	Dimensional coordinates (m)
X, Y	Dimensionless coordinates

Greek Symbols:

α	Thermal diffusivity ($m^2\,s^{-1}$)
β	Thermal expansion coefficient (K^{-1})
φ	Nanoparticle's volume fraction
ν	Kinematic viscosity ($m^2\,s^{-1}$)
θ	Dimensionless temperature
ρ	Density ($kg\,m^{-3}$)
μ	Dynamic viscosity ($N\,s\,m^{-2}$)

Subscripts:

c	cold
f	fluid
h	hot
nf	nanofluid
s	solid particle

Afroza Nahar is an Associate Professor in the Department of Computer Science, American International University-Bangladesh (AIUB), Dhaka - 1229, Bangladesh. Email: afroza@aiub.edu
Salma Parvin is a Professor in the Department of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka-1000, Bangladesh. Email: salpar@math.buet.ac.bd
Md. Hasanuzzaman is an Associate Professor, UMPEDAC, University of Malaya (UM), 50603 Kuala Lumpur, Malaysia. Email: hasan@um.edu.my

I. INTRODUCTION

NATURAL CONVECTION heat transfer is the main heat transfer mechanism in numerous engineering applications like solar collectors, refrigerator, thermal storage, electronic device cooling etc. [1, 2]. The commonly used heat transfer fluid is water that has very low thermal conductivity. Fluids containing nanoparticles could be a potential solution in this regard [3, 4]. Mixing nanoparticles with base fluid water offers better thermal conductivity than pure water [5].

Many researchers have interest in the work of heat transfer in cavities with nanofluid. Parvin et al studied free convection heat transfer in an enclosure with a heated body using nanofluid [6]. Authors observed that higher cooling performance is possible by adding nanoparticles into pure water. A numerical study on the effects of dynamic viscosity and thermal conductivity of nanofluid has done by Ho et al. [7]. Authors worked with Al_2O_3-water nanofluid using square enclosure shape to enhance thermal conductivity and dynamic viscosity. The study of free convective flow heat transfer and entropy generation in an irregular cavity filled with Cu–water nanofluid was studied by Parvin and Chamkha [8]. Numerically Lin and Violi analyzed natural convection heat transfer and fluid flow in a hollow with differentially heated walls surrounded by Al2O3-water nanofluid [9]. Results showed that inclusion of nanoparticles enhances the effect of thermal diffusivity with decrease in Prandtl number. The natural convection of SiO_2-water nanofluid was studied by

Jahanshahi et al. [10] using two different models and both models were recommended by Hamilton and Crosser [11]. Results showed that thermal conductivity increased in both models through the inclusion of nanoparticles. The heat transfer of a heated cylinder enclosed in a square enclosure filled with water–Cu nanofluid for free convention flow was examined numerically by Parvin et al. [12]. Results indicated that heat transfer enhancement can be possible using high viscosity nanofluid. Magnetohydrodynamic nanofluid slip flow in porous media with nonlinear radiation is analyzed by Jashim et al. using finite element method [13]

Efficiency loss occurs from all thermofluidic processes that involve irreversibilities, entropy generation measures the extent of these irreversibilities. Bejan proposed entropy generation minimization (EGM) method to determine the optimal system design characteristics [14, 15], and serves as an effective approach. Khan and Gorla [16] analyzed the exergetic features of heat transfer and fluid flow of natural convection with non-Newtonian fluids over a horizontal plate of a given surface in a porous medium. Oliveski et al. [17] presented a numerical analysis on entropy generation of natural convection in rectangular cavities, where results indicated that the total entropy generation in steady state increase linearly with the aspect ratio and the irreversibility coefficient, and exponentially with the Rayleigh number. Singh et al. [18] performed a theoretical examination of the entropy production in a tube containing AL2O3–water nanofluid with varying diameters. It was shown that there is a diameter that results in the lowest rate of entropy formation for both laminar and turbulent flow.

Few evaluations have been conducted on the natural convection around a cavity obstruction. Sheikhzadeh et al. [19] investigate the influence of the Prandtl number on the continuous magneto-convection around an adiabatic body in the center of a square cavity. Multiple scholars [20–24] have investigated the entropy production of spontaneous convection in square or rectangular cavities. Recently, using nanofluid to generate entropy in square or wavy-wall cavities has attracted attention. Shahi et al. [25] investigated the entropy production generated by free convection inside a square cavity using Cu-water nanofluid. Results showed that increasing nanoparticle volume fraction increased Nusselt number while decreasing entropy generation.

As seen from the above literature review, the heat transfer performance and entropy generation rate of free convection in cavities with regular shapes, numerous studies have been conducted. Most of the research is done on simple geometries such as rectangular or square enclosures and very few works have been carried out with irregular shapes of channels. In this paper, characteristics of free convection heat transfer and entropy generation rate with an L-shaped cavity containing various nanofluid have been studied numerically. The central aim of this work is to investigate the effects of the different nanofluids and the Prandtl number on the streamlines, the isotherm distribution, the mean Nusselt number, the entropy generation, the Bejan number, and the overall entropy generation.

II. PROBLEM FORMULATION

Fig. 1 depicts the annular space between a hot inner body and its enclosure filled with nanofluid. The outer and inner lengths of the cavity are L and L_1 respectively. The internal temperature of the hot wall is maintained at Th and the external temperature of the cold wall is maintained at Tc, while the other two sides are adiabatic.

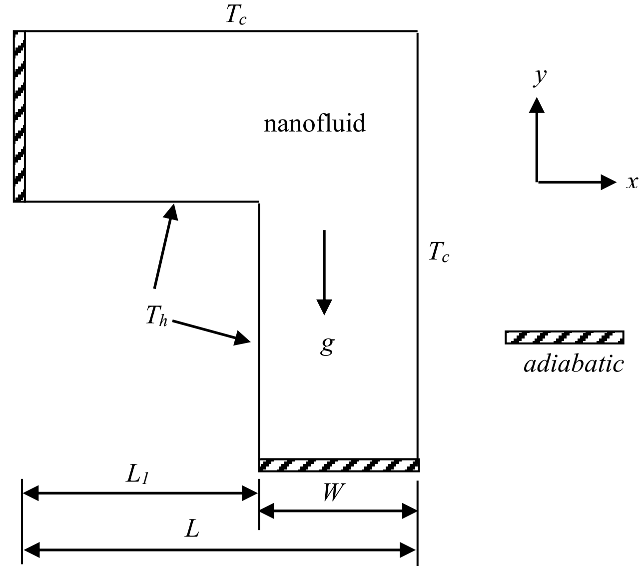


Fig. 1: Geometry of the problem.

III. MATHEMATICAL FORMULATION

For the mathematical modeling of the physical problem, the following assumptions are made:

- 2-D laminar steady and incompressible flow.
- Viscous force and radiation effects are neglected.
- Gravitational force acts along the vertically downward direction.

A. Governing Equations

According to the above assumptions the governing equations under Boussinesq approximation in two dimensional forms are as follows [8]:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations:

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial \rho}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial \rho}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\rho_{nf}\beta_{nf}(T-T_c) \tag{3}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

where, $\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s$ is the density,
$(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s$ is the heat capacitance,
$\beta_{nf}=(1-\varphi)\beta_f+\varphi\beta_s$ is the thermal expansion coefficient,
$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$ is the thermal diffusivity,
$\mu_{nf}=\mu_f(1-\varphi)^{-2.5}$ is dynamic viscosity and
$k_{nf}=k_f\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}$ is the thermal conductivity of nanofluid.

The boundary conditions are:
$T=T_h$ for inside wall
$T=T_c$ for outside walls
$\frac{\partial T}{\partial n}=0$ for the rest of the surfaces
$u=v=0$ for solid boundaries

To make the above equations non-dimensional, parameters are made dimensionless as:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{uL}{\alpha_f},\quad V=\frac{vL}{\alpha_f},\quad P=\frac{pL^2}{\rho_f\alpha_f{}^2},\quad \theta=\frac{T-T_c}{T_h-T_c}$$

Substituting the above variables in equations (1) to (4), the following non-dimensional equations are obtained:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (5)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X}+Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (6)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+RaPr\frac{(1-\varphi)\rho_f\beta_f+\varphi\rho_s\beta_s}{\rho_{nf}\beta_f}\theta \quad (7)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (8)$$

where $Pr=\frac{\nu_f}{\alpha_f}$ is Prandtl number and $Ra=\frac{g\beta_f(T_h-T_c)L^3}{\nu_f\alpha_f}$ is Rayleigh number.

Corresponding boundary conditions takes the following form:

$\theta=1$ for the inside walls,
$\theta=0$ for the outside walls,
$\frac{\partial\theta}{\partial N}=0$ for other surfaces,
$U=V=0$ for all solid boundaries,

The average Nusselt number at the heated surface of the enclosure is:

$$Nu=-\frac{1}{S}\int_0^S\left(\frac{k_{nf}}{k_f}\right)\frac{\partial\theta}{\partial N}dN$$

where $\frac{\partial\theta}{\partial N}=\frac{1}{L}\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$ and S, N are the non-dimensional length and coordinate along the heated surface respectively.

B. Second law formulation

In convection process, the entropy in the fluid is continuously generated due to irreversible nature of heat transfer and effect of viscosity. Entropy generation is a scalar field of temperature and velocity components because it occurs from the heat transfer and fluid friction and responsible for the calculation of degraded energy expressed. The dimensional local entropy generation, S_{gen} is defined as [14]:

$$S_{gen}=\frac{k_{nf}}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\mu_{nf}}{T_0}\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2\right] \quad (9)$$

where $T_0=\frac{T_h+T_c}{2}$.

After substituting dimensionless parameters, the non-dimensional entropy generation (S) becomes:

$$S=S_{gen}\frac{T_0L^3}{k_f(T_h-T_c)^2}=\frac{k_{nf}}{k_f}\left[\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2\right]+\chi\frac{\mu_{nf}}{\mu_f}\left[2\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial V}{\partial Y}\right)^2+\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)^2\right]$$

$$=S_{gen,h}+S_{gen,v} \quad (10)$$

where $S_{gen,h}$ represents the dimensionless entropy generation caused by heat transfer and $S_{gen,v}$ represents the dimensional entropy generation caused by viscous effect. The irreversibility factor χ is defined as:

$$\chi=\frac{T_0\mu_f}{k_f}\frac{U^2}{(T_h-T_c)^2} \quad (11)$$

The Bejan number (Be) is defined as the ratio of the entropy generation caused by heat transfer and the total entropy generation and given by:

$$Be=\frac{S_{gen}}{S} \quad (12)$$

There are three conditions raised for Bejan number (Be): (i) when Be ≈ 1, the heat transfer irreversibility is leading (ii) when Be << 0.5, the irreversibility caused by the viscous effects controls the processes, and (iii) when Be = 0.5, entropy generation caused by the viscous effects and the heat transfer effects are equal.

IV. Numerical Implementation

The non-dimensional governing equations are solved by the Galerkin finite element method with boundary conditions in COMSOL Multiphysics® software and the method yields the subsequent nonlinear residual equations [26]. For mass conservation the continuity equations have been used as a constraint as well as to find the pressure distribution. Ready used the finite element method to solve the Eqs (6) - (8), where the pressure P is eliminated by a constraint [27]. Large values of this constraint are satisfied the continuity equation. Then, using a basis set the velocity components (U, V) and temperature (θ) are extended. For evaluating the integrals in these equations, three points Gaussian quadrature is used. Newton–Raphson method is used to solve the non-linear residual equations for determining the coefficients of the expansions. The error is below that the convergence criteria $|\psi^{n+1} - \psi^{n}| \leq 10^{-4}$, where n is the number of iteration and Ψ is a function of U, V and θ.

A. *Mesh Generation*

Generally, mesh presents the geometric domain on which a problem is solved to make it easy for solution. Figure 2 presents the two-dimensional finite element mesh, mesh primarily of triangular shapes. For this physical domain, free triangular mesh setting is used.

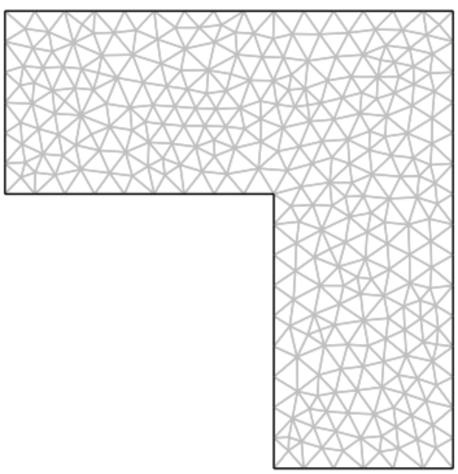


Fig. 2: Finite element mesh

B. *Thermo-physical properties*

The thermo-physical properties of the nanofluid are shown in Table 1.

Table 1. Thermophysical properties of fluid and nanoparticles [28]

Physical Properties	Fluid (water)	Cu	Ag	Al_2O_3
C_p(J/kgK)	4179	385	235	765
ρ (kg/m^3)	997.1	8933	10500	3970
k (W/mK)	0.613	400	429	40
$\alpha \times 10^7$ (m^2/s)	1.47	1163.1	1738.6	131.7

C. *Grid independent test*

To ensure a grid-independent solution, a broad mesh testing procedure is accomplished for Ra = 10^4 and Pr = 6.6 in the given geometry. The simulation is run for highly precise key in the average Nusselt number for Cu-water nanofluid (ϕ = 5%) as well as base fluid water (ϕ = 0%) for 282, 572, 964, 2288 and 6794 number of grid elements. The grid fineness is shown in Fig. 3. Average Nusselt numbers for Cu-water nanofluid and clear water with 2288 elements show very minor discrepancy in results. Therefore, for the computation 2288 elements are chosen as the non-uniform grid system.

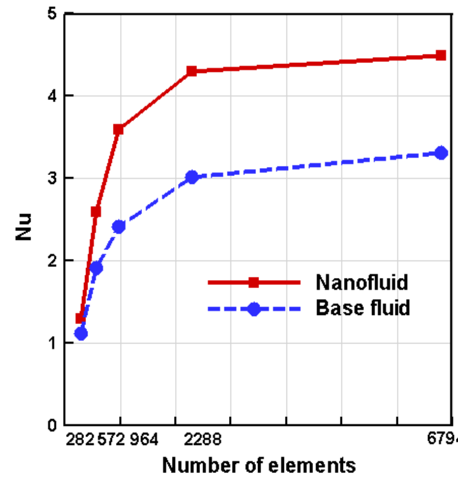


Fig. 3: Grid independent test

V. Result and Discussion

The effect of different parameters is shown in this section. Streamline, entropy generation and isotherms are presented numerically for different nanoparticles: Ag, Cu and Al_2O_3. Prandtl number in an L-shaped cavity also has been presented. The values of Pr are taken 4.2, 6.6, 8.8 and 10.2 [29]. The volume fraction of nanoparticles (φ = 5%) and Rayleigh number (Ra = 10^4) are kept fixed. Moreover, total entropy generation, Bejan number, the mean Nusselt number for heat transfer and entropy generation for viscus term are also presented.

A. *Effect of Prandtl number*

Prandtl number is a physical property of the fluid. Here the considered values of Pr (4.2, 6.6, 8.8 and 10.2) indicate water at different temperatures. That is why the base fluid always remains water. Figures 4 (a) to (c) demonstrate the temperature, flow field, and entropy production due by irreversibility of heat transfer for various Pr for Ag-water nanofluid. Figure 4 (a) shows that the isotherms are almost parallel to the wall of the enclosure at Pr = 4.2, whereas they become significantly twisted for Pr = 10.2. The thermal current activities are much more activated with escalating Pr. The isothermal lines are more condensed at the heat source due to increasing of Pr and a thermal plume took place based on the heated area. The overall heat transfer increases when the temperature distributions become distorted because of increasing Pr. This result can be attributed to control of the viscous force over the thermal force. The thickness of the thermal boundary layer increases near the heated surface with the increase of Prandtl number which points to a steep temperature gradient. As a result, the overall heat transfer increases within the cavity.

In Fig. 4(b), we observe that flow pattern changes significantly with Prandtl number variation. For lower Pr, the fluid flow covers the entire cavity with a vertical and two horizontal swirls. These swirls strengthen with increasing Pr since at higher Prandtl number convection is the dominant mode of heat transfer.

Figure 4(c) is represented the local entropy generation for the Prandtl numbers (Pr) from 4.2 to 10.2, The result shows that lower local entropy generation occurs due to lower value of Pr in the cavity, whereas an increase in Pr causes a greater temperature gradient, resulting in an increase in entropy generation.

B. Effect of different nanoparticles

In Figure 5 (a) to (c), the isothermal lines, streamline and entropy generation for the effect of different nanoparticles and the base fluid water are presented where Pr was 10.2. Figure. 5 (a) shows though isothermal patterns are almost the same for all types of nanofluids as well as base fluid. A careful observation shows that isothermal lines are more twisted for Ag-water nanofluid which indicates increase of the heat transfer coefficient. This happens because of the higher thermal conductivity of Ag nanoparticle than the other considered nanofluids.

From Fig. 5 (b), it can be noticed that a mixed flow structure the vertical vortex which fills a small part of the horizontal extension is also predominant. In the base fluid, the higher dissemination of the vertical vortex into the horizontal chamber is observed. On the other hand, in the case of Ag-water nanofluid, the flow pattern in the horizontal portion exerts a significant effect. In this case, the horizontal part does not allow the vertical flow to enter it. The usual cellular convection pattern is created in the horizontal extension part of the cavity.

Generally, the local entropy generation is occurred due to heat transfer irreversibility From Fig. 5(c), it is seen that slightly higher local entropy generation occurs in the cavity for Ag-water nanofluid compared to other cases due to greater temperature gradient in cavity walls.

C. Nusselt number variation

The variation of average Nusselt number along the hot wall for different Pr and volume fraction of nanoparticles in the nanofluid is shown in Fig. 6 (a) and (b).

From these figures, it is observed that Nu enhances with increase the values of Pr from 4.2 to 10.2 for all nanofluids and base fluid water. As a result, greater Prandtl number causes lower temperature of the fluid which increased the heat transfer rate.

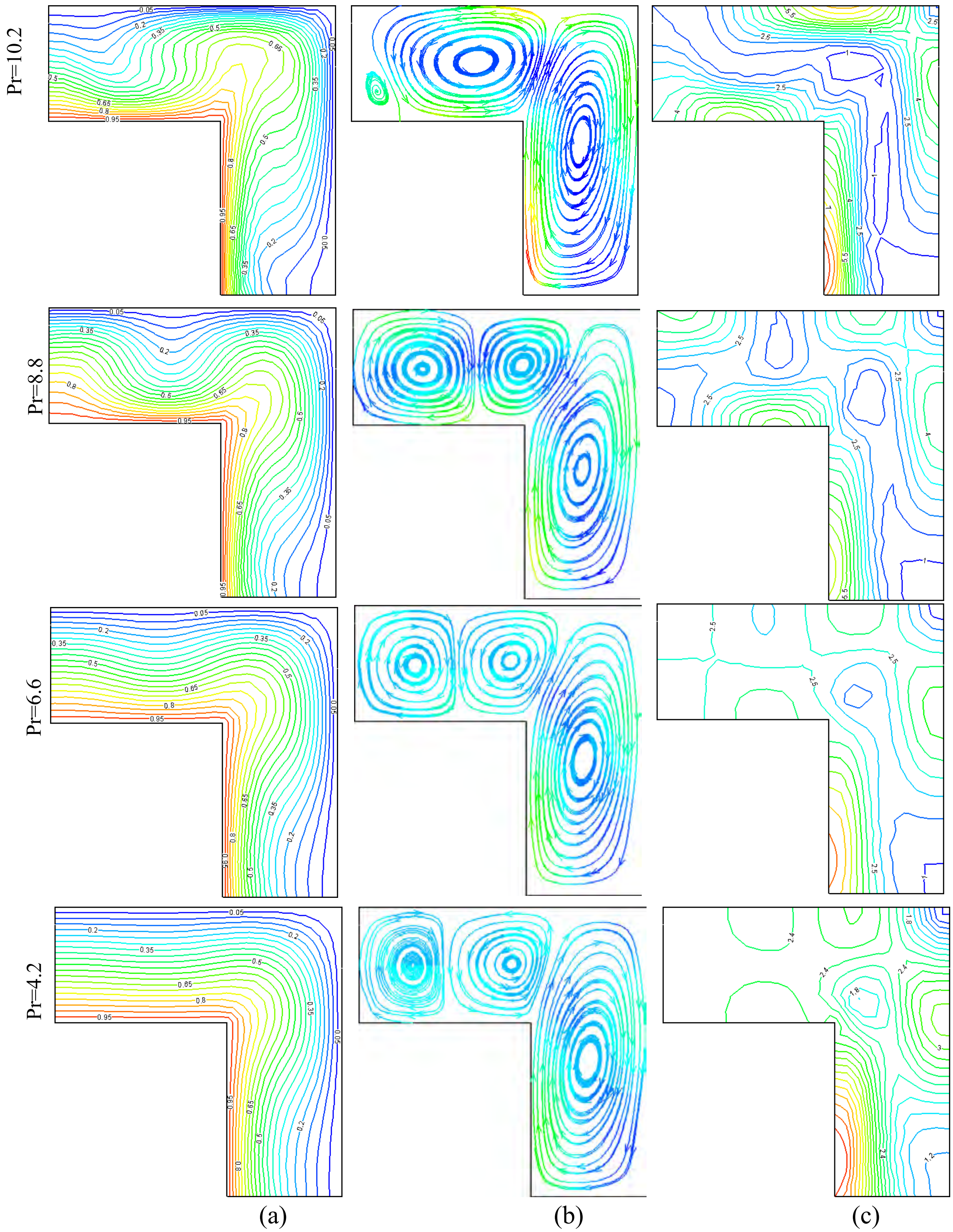


Fig. 4: (a) Isotherms (b) streamlines and (c) entropy generation due to heat transfer for different Prandtl numbers

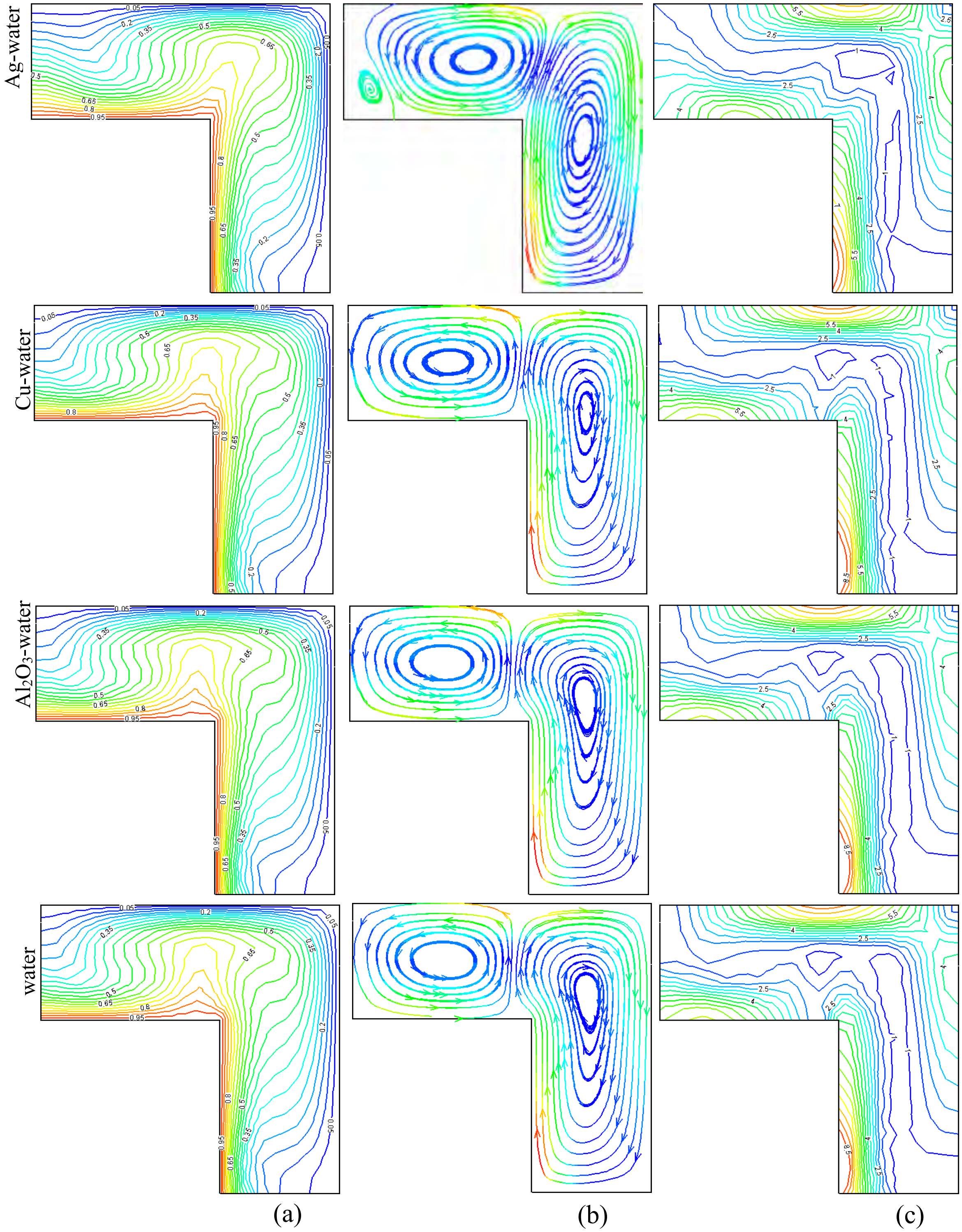
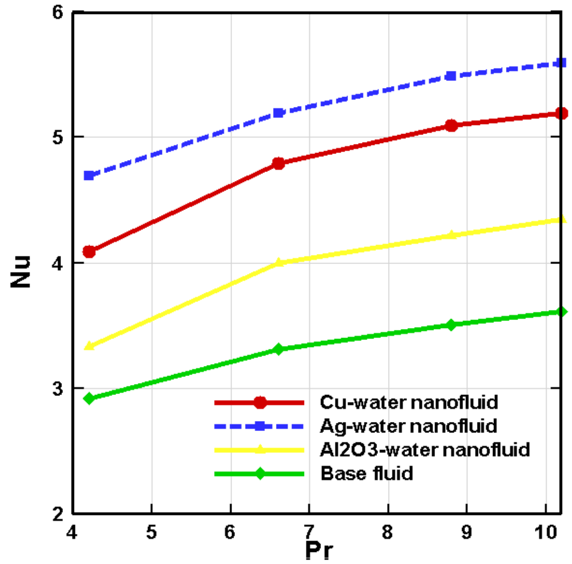
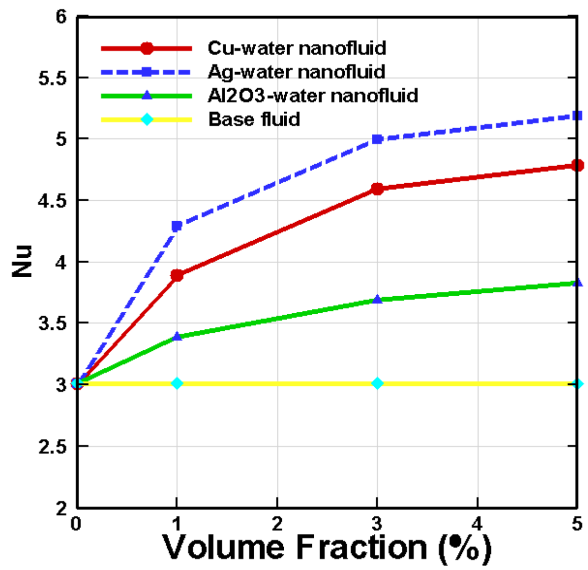


Fig. 5: (a) Isotherms (b) streamlines and (c) entropy due to heat transfer for water and different nanofluids for Pr = 10.2



(a)



(b)

Fig. 6: Average Nusselt number for different nanofluids (a) Pr with φ = 5% and (b) φ with Pr = 10.2

Figure 6 (b) shows Nusselt numbers increase due to the increases of volume fraction for all nanofluids. This is because nanofluids have superior thermal conductivity than water. The maximum heat transfer is observed for Ag-water nanofluid. However, with increase in volume fraction Nu increment rate becomes bland, which manifests a certain limit for enhancement of heat transfer by adding nanoparticle to the pure water.

D. Entropy variation

Fig. 7 (a) to (c) show entropy generation due to heat transfer and viscous effect along with the total entropy generation trend. In Fig. 7 (a), the entropy generation due to heat transfer rises by growing Pr because it creates high temperature gradient. However, Fig. 7 (b) shows that viscosity has relatively less effect on entropy generation. Fig. 7 (c) also confirms the dominance of viscous effects on total entropy generation. However, the effect of Pr is more pronounced in nanofluids than the base fluid in all the forms of entropy. Low Prandtl number in resulting higher thermal conductivity, therefore, fluid temperature decreases when the Prandtl number increases. That is why entropy generation increases during the increase of viscous irreversibility and is more obvious for higher Prandtl number value.

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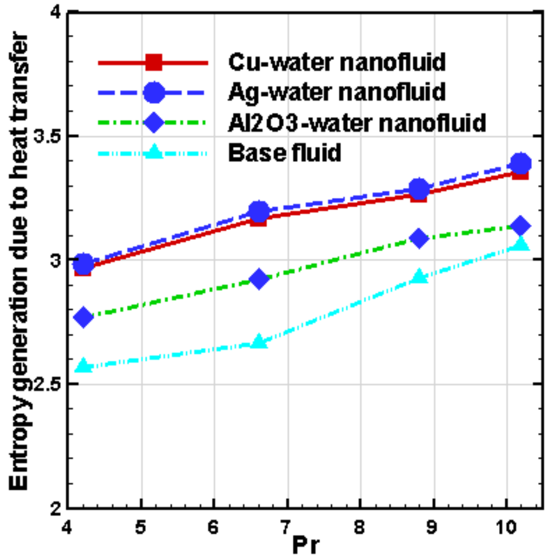
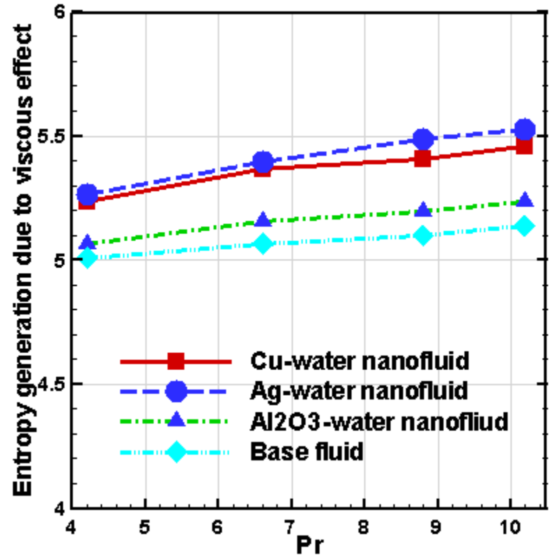
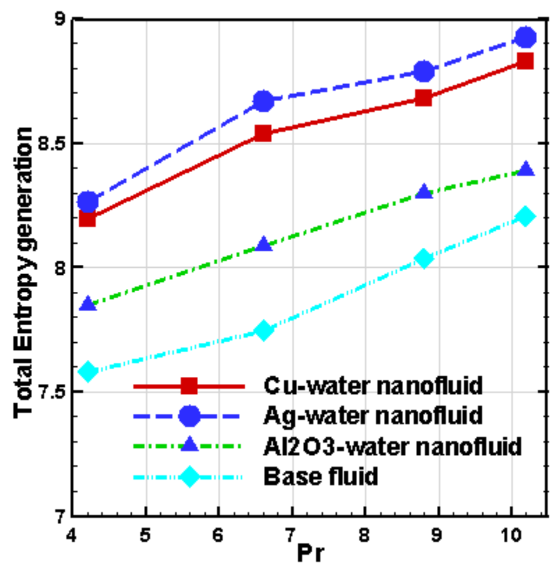


(a)



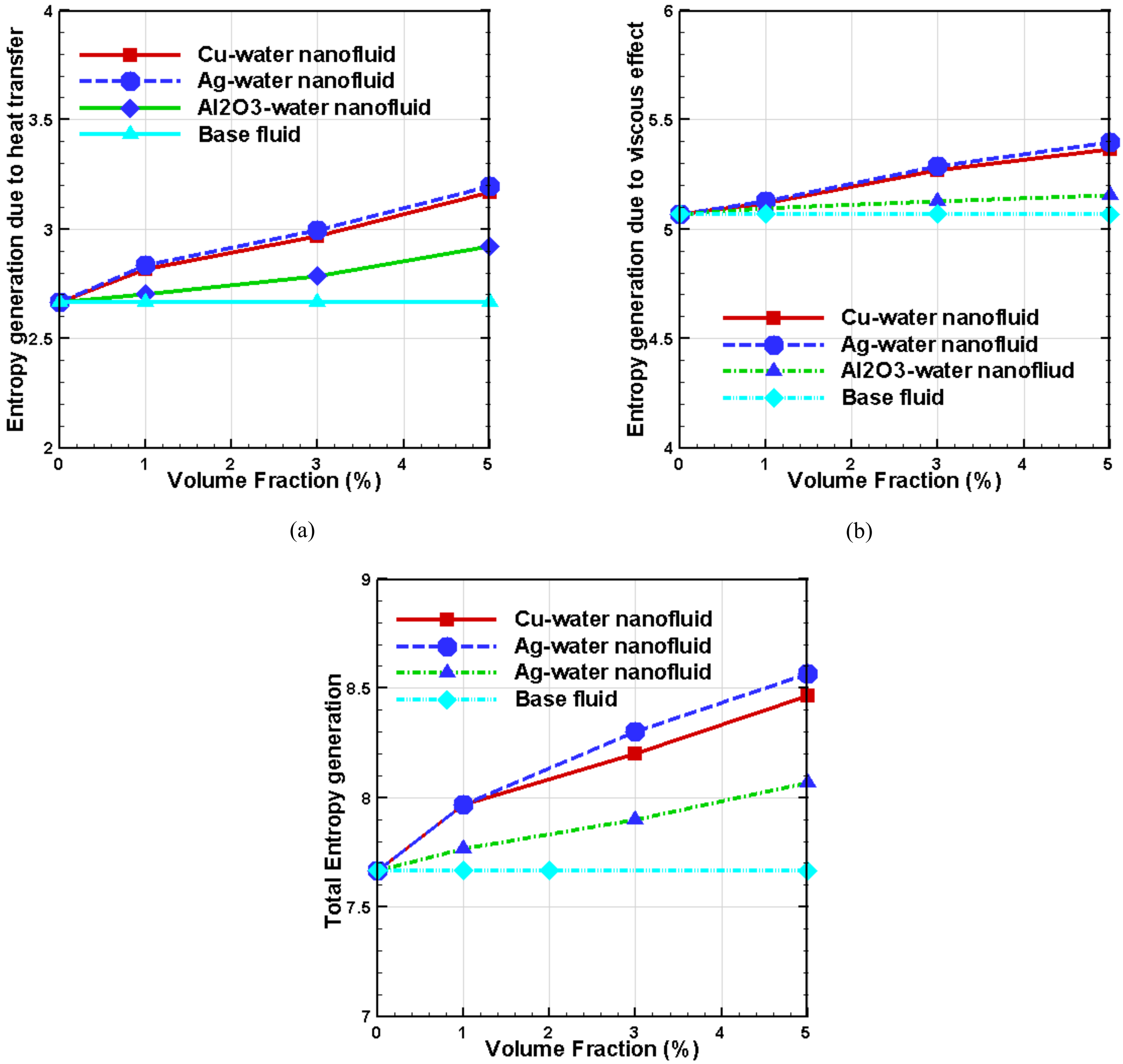
(b)



(c)

Fig. 7: (a) Entropy generation due to heat transfer effects, (b) entropy generation due to viscous effects and (c) total entropy generation for different Pr with φ = 5%

Fig. 8 (a) to (c) show the effect of volume fraction of nanoparticle in nanofluids on entropy generation. It is evident from the figures that all forms of entropy generation increase with volume fraction of nanoparticles in the nanofluid. Entropy generation is greater in the nanofluids than that in base fluid since insertion of nanoparticles creates higher temperature gradient and increases the fluid density that augments the shear forces. Cu-water and Ag-water nanofluid produce more irreversibility than Al_2O_3-water nanofluid and base fluid.



(a) (b)

(c)

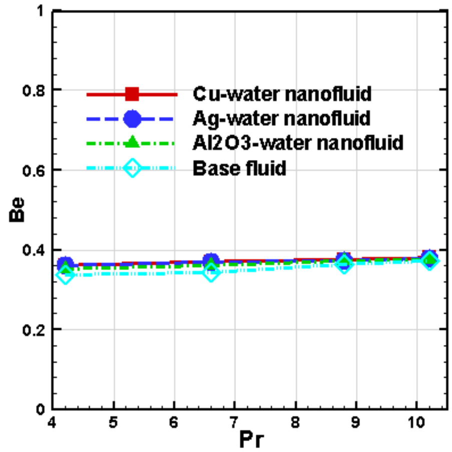
Fig. 8: (a) Entropy generation due to heat transfer effects, (b) entropy generation due to viscous effects and (c) total entropy generation for different nanofluid for Pr = 10.2

E. Bejan number variation

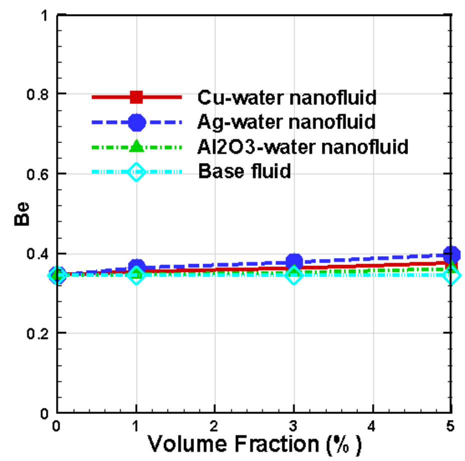
Figure 9 (a) to (b) show the effects of Pr for different volume fractions of different nanoparticles on entropy generation by analyzing the difference of Bejan number Be. It is noticed that values of Be remains less than 0.5 for all considered Prandtl numbers and nanofluids. That is, the viscous effect is pronounced for entropy variation. The viscosity increases by adding nanoparticles with water that increases thermal conductivity.

F. Comparison with previous findings

The average Nusselt number obtained in the present study is equated with those of Nithiarasu et al. [30] for an identical geometry (Table 2). The present results for φ = 0% shows good agreement with the outcomes obtained by Nithiarasu et al. [30].



(a)



(b)

Fig. 9: Bejan number for different nanofluid (a) Pr with $\varphi = 5\%$ and (b) φ with Pr = 10.2

Table 2: Nusselt numbers found in current study and Nithiarasu et al. [30]

Ra	Nithiarasu et al. [30]	Present work for $\varphi = 0\%$	Present work for $\varphi = 5\%$
10^3	3.58	2.89	3.98
10^4	3.59	3.0	4.32
10^5	5.63	5.69	8.18

VI. Conclusion

From the above investigation, it can be concluded that

- Using nanofluids, the entropy generation, Nusselt numbers and the Bejan number increase.
- Nusselt numbers and Bejmin numbers increase when Prandtl numbers increase.
- Minimum total entropy generation is found for base fluid and maximum for Ag- water nanofluid.
- Overall study of different Nusselt numbers and the total entropy generation with change of various parameters involved shows that *Pr* may be an important parameter for desired heat transfer increment with decreasing entropy generation in the given geometry.

In future, the work can be extended for mixed convection phenomena with slip boundary condition using different nanofluids for optimizing heat transfer rate.

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Dr. Afroza Nahar received her PhD degree from University of Malaya, Malaysia. Prior to that she earned a M. Sc in Computer Science from Asian Institute of Technology, Thailand and another in Applied Mathematics from university of Dhaka, Bangladesh. Currently, she is working as an Associate Professor in the Department of Computer Science at American International University-Bangladesh (AIUB). Dr. Nahar published several of her research works in ISI-indexed high impact factor journals and presented many papers in prestigious international conferences. Most of her contributions in Computational modeling of PV/T system Research field were published in ISI Q1 journals. She has attended many national and international conferences in home and abroad. Her major research interests include Computational Modeling, Renewable Energy System, Distributed Computing, Wireless Sensor Network IoT etc. She is a life member of Bangladesh Mathematic Society. She is also joint secretary of Association of Bangladeshi Women in Mathematics (ABWM).

Dr. Salma Parvin is working as a Professor in the Department of Mathematics, Bangladesh University of Engineering and Technology (BUET). She received her PhD and M.Phil. from the Department of Mathematics, Bangladesh University of Engineering and Technology (BUET). She completed her M. Sc in Applied Mathematics and B.Sc (Honors) in Mathematics from the university of Dhaka, Bangladesh. Her research interests include Computational Fluid Dynamics (CFD), Nanofluids Flow Modelling, Numerical Heat Transfer, Magnetohydrodynamics and Quantum Mechanics. She has published more than 100 research papers in recognized and refereed journals/proceedings and 76 conference papers. She has attended many national and international conferences in home and abroad. Dr. Parvin has been serving national and international scientific duties like organizing Undergraduate Mathematics Olympiad in national level and international conference. She is working as the president of Association of Bangladeshi Women in Mathematics (ABWM). She has been chosen as the CWM (Committee for Women in Mathematics of IMU)

Dr. Md. Hasanuzaman is currently working as Associate Professor at the UM Power Energy Dedicated Advanced Centre (UMPEDAC), University of Malaya. He was listed among the World's Top 2% Scientists for the years 2021 & 2020. He is an Associate Editor of the Alexandria Engineering Journal, Elsevier; Associate Editor in Chief of the International Journal of Renewable Energy Resources; Regional Editor of the Journal of Thermal Engineering and has been a Guest Editor for Renewable Energy, Elsevier. His books (i) Technologies for Solar Thermal Energy, 2022 and (ii) Technologies Energy for Sustainable Development 2020 published by Elsevier. Hisresearch interests include thermal engineering, renewable energy, energy and buildings, energy policy, energy and transport, electric vehicles. He has authored and co-authored more than 120 research papers accumulated more than 5,100 and 3890 citations with an h-index of 36 & 33 in the Scopus and WoS indexed respectively. He has secured and managed more than 7 million Ringgit national and international research grants as a PI & co-PI. He received a University of Malaya Excellence Award 2012 for his outstanding achievement in PhD.