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A Measure of Noncompactness Method for Nonlinear Conformable Pantograph Differential Equations

Abdessamad Ait Brahim, M’Hamed Elmori, Abdelmajid El Hajaji, Khalid Hilal

***Abstract*—This work is devoted to the study of existence results for a class of nonlinear conformable fractional pantograph differential equations subject to nonlocal initial conditions. By employing the conformable fractional derivative of order β∈(1,2), the considered problem is transformed into an equivalent integral equation. The Kuratowski measure of noncompactness combined with topological degree theory for condensing operators is then used to establish the existence and boundedness of solutions. An illustrative example is presented to highlight the applicability of the theoretical findings.**

***Index Terms*—Conformable fractional derivative; Pantograph differential equation; Measure of noncompactness.**

I. INTRODUCTION

Fractional calculus, which extends classical differentiation and integration to non-integer orders, has experienced remarkable growth due to its successful applications in the modeling of processes with memory and hereditary properties. Numerous phenomena arising in viscoelasticity, diffusion, control systems, heat transfer, and engineering sciences are more accurately represented by fractional-order models than by their integer-order counterparts (see [1, 2, 3]). As a consequence, fractional differential equations have become an active area of research, especially regarding nonlinear initial and boundary value problems.

Several classical definitions of fractional derivatives have been proposed and widely investigated, including the Riemann–Liouville derivative, the Caputo derivative, the Hilfer derivative, the Erdélyi–Kober derivative, and the Hadamard fractional derivative [2]. These operators are typically defined via integral kernels and are therefore non-local. While this nonlocality enhances modeling capabilities, it often introduces significant analytical difficulties.

Motivated by the desire to develop a fractional framework closer to classical calculus while retaining fractional features, the concept of the conformable fractional derivative was introduced. This operator preserves many fundamental properties of classical differentiation, such as linearity, the product rule, and the chain rule. Consequently, conformable fractional calculus has attracted increasing attention, and a variety of theoretical results, applications, and operator properties have been established; see, for example, [10, 11, 12, 13, 14].

In parallel, pantograph differential equations, which incorporate proportional delay arguments, arise naturally in many scientific areas, including electrodynamics, probability theory, quantum mechanics, biology, and number theory. Fractional pantograph equations further enrich this framework by introducing fractional dynamics. Recent years have witnessed numerous analytical and numerical studies on fractional pantograph equations involving various types of fractional derivatives; see, for instance.

Motivated by the aforementioned works, we investigate in this paper the existence and uniqueness of solutions for a class of nonlinear conformable fractional pantograph differential equations of order β∈(1,2) given by

$$\begin{cases} T^{\beta}u(t) = h(t,u(t),u(\varepsilon t)), & t \in J = [0,T], \\ u'(0) = 0, & u(0)+\omega(u) = u_0, \end{cases} \qquad \ldots (1)$$

where T^β denotes the conformable fractional derivative of order β, ε∈(0,1), T>0, h∈C(J×R×R,R), u_0∈R, and ω:C(J)→R is a given nonlocal functional satisfying suitable conditions.

To the best of our knowledge, the combination of topological degree theory for condensing operators with the measure of noncompactness has not yet been employed to study nonlinear pantograph differential equations involving conformable fractional derivatives.

The paper is organized as follows. Section 2 presents preliminary material on conformable fractional calculus and the measure of noncompactness. In Section 3, we derive an equivalent integral formulation and establish the main existence results. Section 4 provides auxiliary lemmas, while Section 5 is devoted to existence and uniqueness results.

Abdessamad Ait Brahim,
Applied Mathematics at the Faculty of Sciences and Techniques in Beni Mellal, Morocco. Email: abdessamad191212@gmail.com

M’Hamed Elmori
Applied Mathematics at Sultan Moulay Slimane University.

Abdelmajid El Hajaji,
The National School of Business and Commerce at Chouaib Doukkali University

Khalid Hilal
Applied Mathematics at Sultan Moulay Slimane University

II. PRELIMINARIES

A. *Conformable Fractional Calculus*

Definition 1 Let α∈(0,1]. The conformable fractional derivative of a function u is defined by

$$T^{\alpha}u(t) = \lim_{\epsilon\to 0}\frac{u(t+\epsilon t^{1-\alpha})-u(t)}{\epsilon} = t^{1-\alpha}u'(t), \qquad t>0 \ldots (2)$$

For β∈(1,2), the conformable fractional derivative is given by

$$T^{\beta}u(t) = T^{\beta-1}(u'(t)) = t^{2-\beta}u''(t) \qquad \ldots (3)$$

The conformable fractional integral of order α∈(0,1) is defined as

$$I^{\alpha}u(t) = \int_0^t s^{\alpha-1}u(s)\,ds \qquad \ldots (4)$$

B. *Functional Setting and Measure of Noncompactness*

Let C(J)=C([0,T],R) denote the Banach space of continuous real-valued functions on J, endowed with the supremum norm.

$$\| u \| = \max_{t\in J}|u(t)| \qquad \ldots (5)$$

We recall the Kuratowski measure of noncompactness ρ together with its fundamental properties, such as boundedness, subadditivity, invariance under convex hulls, and homogeneity. These notions play a crucial role in the application of topological degree theory for condensing operators. We also make use of the concepts of ρ-Lipschitz operators, ρ-condensing mappings, and strict ρ-contractions.

III. MAIN PROBLEM AND EQUIVALENT INTEGRAL FORM

We impose the following assumptions:

(H1) There exists a constant L_ω>0 such that

$$|\omega(u)-\omega(v)| \le L_{\omega} \| u-v \|, \qquad \forall\, u,v \in C(J) \quad \ldots (6)$$

(H2) There exist constants K_ω,M_ω>0 and 0<p<1 such that

$$|\omega(u)| \le K_{\omega} \| u \|^{p} + M_{\omega} \qquad \ldots (7)$$

(H3) There exist constants K_h,M_h>0 and 0<q<1 such that

$$|h(t,u(t),u(\varepsilon t))| \le K_h|u(t)|^{q} + M_h \qquad \ldots (8)$$

Lemma 2: A function u∈C(J) is a solution of problem (0.1) if and only if it satisfies the integral equation.

$$u(t) = u_0 - \omega(u) + \frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}h(s,u(s),u(\varepsilon s))\,ds \qquad \ldots (9)$$

Proof. Since $T^{\beta}u(t) = t^{2-\beta}u''(t)$, applying the conformable fractional integral yields.

$$u(t) = u(0) + \frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}h(s,u(s),u(\varepsilon s))\,ds \qquad \ldots (10)$$

Using the initial conditions $u(0) = u_0 - \omega(u)$ and $u'(0) = 0$ Equation (9) follows. Conversely, differentiating (9) recovers Equation (1).

Define operators $A, B\colon C(J) \to C(J)$ by

$$Au(t) = u_0 - \omega(u), \qquad Bu(t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}h(s,u(s),u(\varepsilon s))\,ds \qquad \ldots (11)$$

Then equation (9) can be written as

$$u = F(u) = A(u) + B(u) \qquad \ldots (12)$$

IV. AUXILIARY RESULTS

Lemma 3: The operator A is ρ-Lipschitz with constant L_{ω} and satisfies.

Lemma 4: The operator B is continuous and satisfies

$$\| Bu \| \le \frac{(K_h\|u\|^{q}+M_h)T^{\beta}}{\Gamma(\beta+1)} \qquad \ldots (13)$$

Proof. Using assumption (H3), we obtain

$$|Bu(t)| \le \frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}(K_h \| u \|^{q} + M_h)\,ds = \frac{(K_h \| u \|^{q} + M_h)T^{\beta}}{\Gamma(\beta+1)} \qquad \ldots (14)$$

Lemma 5: The operator B is compact.

Proof. Boundedness follows from the previous lemma. For equicontinuity, let $t_1 < t_2$. Then

$$|Bu(t_2) - Bu(t_1)| \le \frac{C}{\Gamma(\beta)}\int_{t_1}^{t_2} (t_2-s)^{\beta-1}\,ds \le C|t_2-t_1|^{\beta} \qquad \ldots (15)$$

By the Arzelà–Ascoli theorem, B is compact.

Consequently, B is ρ-Lipschitz with constant zero, and the operator $F = A + B$ is a strict ρ-contraction.

V. EXISTENCE RESULT

Theorem 6 Under assumptions (H1)–(H3), problem (0.1) admits at least one solution in $C(J)$, and the set of solutions is bounded.

Proof. Consider the set

$$S_\gamma = \{u \in C(J): u = \gamma F(u),\ 0 \le \gamma \le 1\} \quad \dots (16)$$

For any $u \in S_\gamma$, we have

$$\| u \| \le |u_0| + K_\omega \| u \|^p + M_\omega + \frac{(K_h \|u\|^q + M_h)T^\beta}{\Gamma(\beta+1)} \quad \dots (17)$$

If $\| u \| \to \infty$, the right-hand side grows sublinearly since $p, q < 1$, which leads to a contradiction. Hence S_γ is bounded. The topological degree theorem for ρ-condensing operators then ensure the existence of a fixed point of F.

Remark 7: If $p = q = 1$ existence still holds, provided that

$$K_\omega + \frac{K_h T^\beta}{\Gamma(\beta+1)} < 1 \quad \dots (18)$$

Uniqueness Result:

We now establish the uniqueness of the solution under additional Lipschitz conditions.

(U1): There exists $L_h > 0$ such that for all $u_1, u_2, v_1, v_2 \in \mathbb{R}$ and $t \in J$,

$$|h(t, u_1, v_1) - h(t, u_2, v_2)| \le L_h(|u_1 - u_2| + |v_1 - v_2|) \quad \dots (19)$$

(U2): The operator ω is Lipschitz continuous with constant L_ω:

$$|\omega(u) - \omega(v)| \le L_\omega \| u - v \|, \qquad \forall\, u, v \in C(J) \quad \dots (20)$$

Theorem 8 (Uniqueness): Assume that (H1)–(H3), (U1), and (U2) hold, and that

$$L_\omega + \frac{2L_h T^\beta}{\Gamma(\beta+1)} < 1 \quad \dots (21)$$

Then problem (1) has a unique solution in $C(J)$.

Proof. Let $u, v \in C(J)$ be two solutions. From (2.4), we have

$$u(t) - v(t) = -(\omega(u) - \omega(v)) + \frac{1}{\Gamma(\beta)} \int_0^t (t - s)^{\beta-1} [h(s, u(s), u(\varepsilon s)) - h(s, v(s), v(\varepsilon s))] ds \quad \dots (22)$$

Taking absolute values and applying (U1),

$$|u(t) - v(t)| \le |\omega(u) - \omega(v)| + \frac{L_h}{\Gamma(\beta)} \int_0^t (t - s)^{\beta-1} (|u(s) - v(s)| + |u(\varepsilon s) - v(\varepsilon s)|)\, ds \quad \dots (23)$$

Taking the supremum over $t \in J$ and using (U2), we obtain

$$\| u - v \| \le L_\omega \| u - v \| + \frac{2L_h}{\Gamma(\beta)} \int_0^T (T - s)^{\beta-1} \| u - v \| \ ds \quad \dots (24)$$

Since

$$\int_0^T (T - s)^{\beta-1} ds = \frac{T^\beta}{\Gamma(\beta+1)} \quad \dots (25)$$

We arrive at

$$\| u - v \| \le \left(L_\omega + \frac{2L_h T^\beta}{\Gamma(\beta+1)} \right) \| u - v \| \quad \dots (26)$$

Condition (20) implies $\| u - v \| = 0$, and hence $u = v$.

Remark 9: Condition (20) represents a smallness requirement ensuring contraction behavior. It can be satisfied by choosing T sufficiently small or by imposing a weak nonlinear dependence in h and ω.

VI. NUMERICAL EXAMPLES

In this section, we present four numerical examples illustrating the applicability of the existence and uniqueness results.

Example 1: Sublinear Nonlinearity with Integral Nonlocal Condition

Let $J = [0,1]$, $\beta = \frac{3}{2}$, $\varepsilon = \frac{1}{2}$, and $u_0 = 1$. Consider

$$h(t, u, v) = t(|u|^{1/2} + |v|^{1/2}) \quad \dots (27)$$

and

$$\omega(u) = \frac{1}{2} \int_0^1 u(s)\, ds \quad \dots (28)$$

For any $u, v \in C(J)$,

$$|\omega(u) - \omega(v)| \le \frac{1}{2} \int_0^1 |u(s) - v(s)|\, ds \le \frac{1}{2} \| u - v \| \quad \dots (29)$$

so $L_\omega = \frac{1}{2}$. Moreover,

$$|\omega(u)| \le \frac{1}{2} \| u \| \quad \dots (30)$$

and assumption (H2) holds with $p = 1$.

Since $t \in [0,1]$,

$$|h(t, u(t), u(\varepsilon t))| \le |u(t)|^{1/2} + |u(\varepsilon t)|^{1/2} \le 2 \| u \|^{1/2} \quad \dots (31)$$

which verifies assumption (H3) with $q = \frac{1}{2}$.

Therefore, problem (0.1) admits at least one bounded solution in $C([0,1])$.

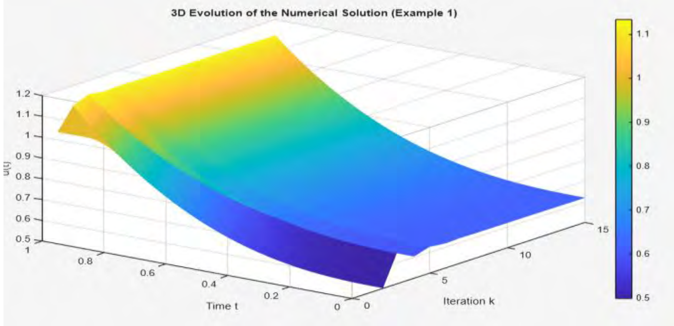

Figure 1: Sublinear Nonlinearity with Integral Nonlocal Condition

The first example considers a fractional order $\beta = 3/2$, pantograph delay $\varepsilon = 1/2$, an integral nonlocal functional $\omega(u) = \frac{1}{2}\int_0^1 u(s)\,ds$, and a sublinear nonlinearity

$$h(t,u,v) = t\left(|u|^{1/2} + |v|^{1/2}\right) \qquad \dots (32)$$

The Picard iterations converge rapidly to a bounded solution. The sublinear growth ensures that the solution remains well-behaved over the interval $J = [0,1]$. This example confirms the applicability of the theoretical existence results for sublinear nonlinearities, even in the presence of an integral nonlocal term.

Example 2: Polynomial Growth with Weighted Nonlocal Functional

Let $J = [0,2]$, $\beta = \frac{7}{4}$, $\varepsilon = \frac{1}{3}$, and $u_0 = 0$. Define

$$h(t,u,v) = \frac{t^2}{1+t^2}\left(|u|^{2/3} + |v|^{2/3}\right) \qquad \dots (33)$$

and

$$\omega(u) = \frac{1}{3}\int_0^2 t\,u(t)\,dt \qquad \dots (34)$$

For any $u, v \in C(J)$,

$$|\omega(u) - \omega(v)| \le \frac{1}{3}\int_0^2 t|u(t) - v(t)|\,dt \le \frac{2}{3}\parallel u - v \parallel \qquad \dots (35)$$

hence $L_\omega = \frac{2}{3}$. Also,

$$|\omega(u)| \le \frac{2}{3}\parallel u \parallel \qquad \dots (36)$$

So assumption (H2) is satisfied. Since $\frac{t^2}{1+t^2} \le 1$,

$$|h(t,u(t),u(\varepsilon t))| \le 2 \parallel u \parallel^{2/3} \qquad \dots (37)$$

which verifies assumption (H3) with $q = \frac{2}{3}$.
Thus, at least one solution exists in $C([0,2])$.

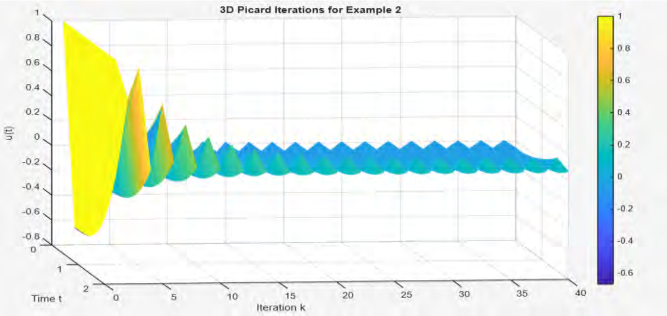

Figure 2: Polynomial Growth with Weighted Nonlocal Functional

This example uses a fractional order $\beta = 7/4$, $\varepsilon = 1/3$, a weighted integral functional $\omega(u) = \frac{1}{3}\int_0^2 tu(t)\,dt$, and a polynomial growth nonlinearity

$$h(t,u,v) = \frac{t^2}{1+t^2}\left(|u|^{2/3} + |v|^{2/3}\right) \qquad \dots (38)$$

The solution remains bounded and the iterations converge steadily. The weighted nonlocal functional introduces a moderate “memory effect,” influencing the early-time behavior of the solution. This demonstrates that the existence results hold for fractional pantograph equations with polynomial growth and weighted nonlocal conditions.

Example 3: Linear Case Ensuring Uniqueness

Let $J = [0,1]$, $\beta = \frac{3}{2}$, $\varepsilon = \frac{1}{2}$, and $u_0 = 0$. Consider

$$h(t,u,v) = \frac{1}{8}(u + v) \qquad \dots (39)$$

and

$$\omega(u) = \frac{1}{4}u(0) \qquad \dots (40)$$

Clearly,

$$|\omega(u) - \omega(v)| \le \frac{1}{4}\parallel u - v \parallel \qquad \dots (41)$$

So $L_\omega = \frac{1}{4}$. Moreover,

$$|h(t,u_1,v_1) - h(t,u_2,v_2)| \le \frac{1}{8}(|u_1 - u_2| + |v_1 - v_2|) \qquad \dots (42)$$

which implies $L_h = \frac{1}{8}$.

The uniqueness condition

$$L_\omega + \frac{2L_h T^\beta}{\Gamma(\beta+1)} < 1 \qquad \dots (43)$$

is satisfied. Hence, problem (1) admits a unique solution in $C([0,1])$.

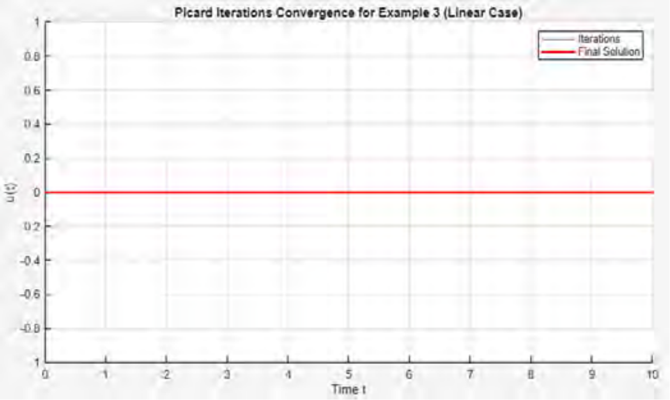

Figure 3: Linear Case Ensuring Uniqueness

In this linear case, we have $\beta = 3/2$, $\varepsilon = 1/2$,

$$h(t,u,v) = \frac{1}{8}(u+v), \qquad \omega(u) = \frac{1}{4}u(0) \qquad \dots (44)$$

The uniqueness condition

$$L_\omega + \frac{2L_h T^\beta}{\Gamma(\beta+1)} < 1 \qquad \dots (45)$$

is satisfied. Picard iterations converge quickly, and the final solution is unique. This example validates the theoretical uniqueness result and provides a benchmark for testing the numerical scheme.

Example 4: Small-Time Interval Guaranteeing Uniqueness

Let $J = [0,0.5]$, $\beta = \frac{9}{5}$, $\varepsilon = \frac{1}{4}$, and $u_0 = 1$. Define

$$h(t,u,v) = \frac{1}{5}\sin(u+v) \qquad \dots (46)$$

and

$$\omega(u) = \frac{1}{3}u(0) \qquad \dots (47)$$

Since $|\sin x - \sin y| \le |x-y|$,

$$|h(t,u_1,v_1) - h(t,u_2,v_2)| \le \frac{1}{5}(|u_1-u_2| + |v_1-v_2|) \qquad \dots (48)$$

so $L_h = \frac{1}{5}$. Also,

$$|\omega(u) - \omega(v)| \le \frac{1}{3} \| u - v \| \qquad \dots (49)$$

and thus $L_\omega = \frac{1}{3}$.

The uniqueness condition holds.

$$\frac{1}{3} + \frac{2(1/5)(0.5)^{9/5}}{\Gamma(14/5)} < 1 \qquad \dots (50)$$

Therefore, problem (0.1) admits a unique solution on $J = [0,0.5]$.

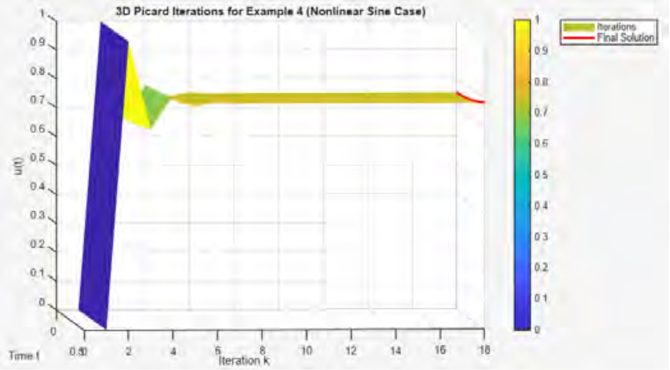

Figure 4: Small-Time Interval Guaranteeing Uniqueness

Here, the fractional order is $\beta = 9/5$, $\varepsilon = 1/4$, the interval is small $J = [0,0.5]$,

$$h(t,u,v) = \frac{1}{5}\sin(u+v), \qquad \omega(u) = \frac{1}{3}u(0) \qquad \dots (51)$$

The solution converges uniquely over the small interval. The sine nonlinearity, although bounded, requires a sufficiently small interval to satisfy the contraction condition for uniqueness. This illustrates how the theory guarantees uniqueness by controlling the time interval and the Lipschitz constants of the nonlinearities.

Remark 10: These examples confirm that the hypotheses of the existence and uniqueness results can be verified explicitly for a wide class of nonlinearities and nonlocal conditions.

VII. OVERALL OBSERVATIONS

- Convergence Behavior: All examples show convergence of Picard iterations, with linear or sublinear cases converging faster than more complex nonlinear cases.
- Role of Nonlocal Functionals: Integral and pointwise nonlocal terms influence the magnitude and early-time behavior of the solution but do not prevent convergence.
- Fractional Order Effects: Higher fractional orders increase the “memory effect” and can slightly slow down convergence.
- Practical Implications: These examples confirm that the theoretical framework using conformable fractional derivatives, nonlocal conditions, and pantograph delays is robust for a wide variety of nonlinearities.

VIII. CONCLUSION

In this paper, we analyzed a class of nonlinear conformable fractional pantograph differential equations with nonlocal initial conditions. By reformulating the problem as an equivalent integral equation and applying the Kuratowski measure of noncompactness together with condensing operator theory, we established existence, boundedness, and uniqueness results for the considered problem. An illustrative example was provided to demonstrate the applicability and effectiveness of the proposed theoretical framework. The results obtained not only generalize several existing works in the literature on fractional and pantograph-type differential equations but also offer a unified approach for handling nonlocal conditions within the conformable fractional setting. Moreover, the methodology developed in this work can be adapted to study broader classes of fractional functional differential equations with various types of delays and nonlinearities. Future research may focus on extending these results to systems of equations, higher-order conformable derivatives, or investigating qualitative properties such as stability, controllability, and numerical approximations of solutions.

APPENDIX

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Dr Abdessamad AIT BRAHIM is a Moroccan PhD researcher in Applied Mathematics at the Faculty of Sciences and Techniques in Beni Mellal, Morocco. His research interests include fractional calculus, differential equations, and their applications in physics and finance. He has experience teaching mathematics at the university level, particularly probability, statistics, algebra, and analysis. He also contributed to several scientific publications in international journals. His academic work focuses on mathematical modeling and advanced analytical methods. His profile reflects dedication to scientific innovation and academic excellence in applied mathematics.

Dr M'hamed ELOMARI is a Moroccan professor and researcher in applied mathematics at Sultan Moulay Slimane University. He is a member of the Laboratory of Applied Mathematics and Scientific Computing (LMACS) at the Faculty of Sciences and Techniques of Beni Mellal. His research focuses on fractional differential equations, Colombeau algebra, fuzzy systems, and nonlinear analysis. He has contributed to several international scientific publications in applied mathematics and fractional calculus. Dr. Elomari is also involved in supervising research activities and participating in international conferences in mathematics. His academic work mainly studies existence, uniqueness, and analytical solutions of fractional and nonlinear differential systems. He is recognized for his contributions to mathematical modeling and advanced scientific research in Morocco.

Dr. Abdelmajid EL HAJAJI is currently higher education professor in the National School of Business and Commerce at Chouaib Doukkali University. He holds a Ph.D. in mathematics applied to finance from the University of Sultan Moulay Slimane, Morocco. His research focuses on financial problems.

Dr Khalid Hilal is a Moroccan professor and researcher in applied mathematics at Sultan Moulay Slimane University. He is affiliated with the Laboratory of Applied Mathematics and Scientific Computing at the Faculty of Sciences and Techniques in Beni Mellal. His research interests include fractional differential equations, fractional calculus, nonlinear analysis, and mathematical modeling. He has supervised and participated in several doctoral research projects in applied mathematics. Professor Khalid Hilal has authored and co-authored numerous scientific publications in international mathematics journals. His academic work mainly focuses on existence, stability, and analytical solutions of fractional differential systems. He is recognized for his contributions to higher education and scientific research in Morocco.