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# Advancing Fuzzy Logic: A Hierarchical Fuzzy System Approach

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***Abstract*—Fuzzy logic systems (FLS) are widely used in various engineering, medical, and scientific applications for modelling complex and uncertain systems. However, traditional FLS has limitations in handling complex and hierarchical structures due to their lack of scalability and interpretability. This paper proposes an approach to hierarchical fuzzy systems (HFS) that enhance the traditional FLS by providing a hierarchical structure with multiple levels of fuzzy rules. The main contribution of this paper is the proposal of HFS, which improves interpretability, scalability, and accuracy compared to traditional FLS, particularly for real-world applications. However, the question arises, "How can the FLS be converted into the HFS?" In this paper, the approach to HFS architecture will comprise two levels of FLS, where the the first level determines the overall behaviour of the system, and the second level refines the output by considering the local behaviour. The proposed approach has been validated through experimental results in a case study, such as the Iris flower classification. The results demonstrate that HFS provides more efficient and reliable solutions and can be applied to various complex and hierarchical systems in different domains, such as manufacturing, robotics, and decision-making.**

***Index Terms*—Fuzzy Logic System, Hierarchical Fuzzy Systems, Interpretability, Complexity**

## I. Introduction

THE Fuzzy Logic Systems (FLS) have become popular in various fields, including engineering, medicine, and science, because of their ability to model complex and uncertain systems [1].

However, traditional FLS has limitations when dealing with complex and hierarchical structures, primarily due to scalability and interpretability issues [2]. To overcome these limitations, this paper proposes a novel approach called Hierarchical Fuzzy Systems (HFS), which enhances traditional FLS by introducing a hierarchical structure with multiple levels of fuzzy rules [3].

The main contribution of this paper is to compare HFS and traditional FLS comprehensively. By comparing these two approaches, the paper aims to demonstrate the advantages of HFS over FLS in terms of interpretability, scalability, and accuracy, particularly in real-world applications.

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Through a thorough comparison between HFS and FLS, this paper aims to contribute to understanding the benefits and limitations of each approach. The findings of this comparison will provide valuable insights for researchers and practitioners when selecting the most suitable methodology for complex and hierarchical systems in different domains, such as manufacturing, robotics, and decision-making.

## II. Literature

This section is intended to offer essential background information about the study subjects. It highlights knowledge gaps between FLS and HFS, particularly regarding their functionalities, benefits and existing challenges.

### A. Fuzzy Logic System

FLS was initially introduced by Lotfi Zadeh in 1965 [4], a Professor at the University of California, Berkeley. He introduced the notion of fuzzy sets to handle such uncertain environments and vagueness in information, which now forms the basis for modern control theory. For over fifty years, FLSs provided the basis for a successful method of modelling uncertainty, vagueness and imprecision, particularly in consumer products and control applications [5], [6].

*1) How it works – FLS:* The principle of fuzzy logic is to formalize and mechanize two essential human skills. Firstly, it enables reasoning and decision-making in environments with ambiguity, imprecision, incomplete information, contradictory information, and partial truth or possibility. Secondly, it allows for performing various tasks without precise measurements or computations [6]. Fuzzy logic can be described as a logic of approximation characterized by fuzzy truth values expressed in linguistic terms, imprecise truth tables, and inference rules with approximate validity [7].

FLSs are commonly used to represent nonlinear, unpredictable, and complex systems. Partitioning the system variable space into fuzzy regions using fuzzy sets is a key feature of FLSs [8]. Each region is associated with a rule that describes the system's properties. FLSs consist of a rule base with rules connected to specific areas where the available information is easily understandable. This property has applications in various fields, including medicine, engineering, decision support, and pattern recognition [9]–[12]. FLSs use fuzzy rules that employ linguistic variables and terms close to natural human language. The number of fuzzy rules required grows exponentially with the size of the input space [8]. To form a complete fuzzy system with n input variables and m membership functions for each input variable, $m^n$ rules are needed.

*2) Advantages of Fuzzy Logic System:* Additionally, FLSs have gained acceptance as a methodology for designing robust controllers that can effectively handle uncertainty and imprecision [13], [14]. They have been applied to various problems and have demonstrated their ability to generate more resilient and cost-effective solutions in the face of uncertainties. The interpretability of FLSs is often highlighted as one of their strengths, as they enable the modelling and processing of linguistic variables and rules that are easily understandable [15]. This interpretability is particularly valuable in applications such as knowledge extraction and decision support [16]–[18].

The concept of fuzzy logic is analogous to human experience and inference. Unlike traditional point-to-point control, fuzzy logic control operates on a range-to-point or range-to-range basis. A fuzzy controller produces its output by fuzzifying inputs and outputs using appropriate membership functions. A crisp input is transformed into the various members of the linked membership functions based on its value. From this perspective, the output of a fuzzy logic controller is determined by its memberships in different membership functions, which can be seen as a set of inputs [19]. Since the 1980s, fuzzy logic implementations have been reported in various industries, such as manufacturing, automatic control, vehicle production, banking, hospitals, libraries, and academic education. Fuzzy logic approaches are widely used in today's society.

*3) Current issues:* FLSs have been successfully applied in various domains, particularly in uncertain situations and imprecise information. However, a key challenge with FLSs is that as the number of variables increases, the number of rules grows exponentially [20], [21]. Essentially, the complexity of an FLS escalates exponentially with the number of variables involved. This phenomenon, known as the '*curse of dimensionality*', was identified by Bellman [22] and further examined by Zeng and Keane, [23] from multiple perspectives.

(a) *Rule dimensionality:* The number of rules in the fuzzy rule base increases exponentially with the number of input variables.
(b) *Parameter dimensionality:* The total number of parameters in the mathematical formulas of fuzzy systems increases exponentially with the number of input variables.
(c) *Data or information dimensionality:* The number of data or knowledge sets required to identify fuzzy systems increases exponentially with the number of input variables.

To address this problem, several methods have been proposed for optimizing the size of the rule base, such as rule selection [24], [25], feature selection [26], rule interpolation [27], singular-value decomposition-QR [28], evolutionary algorithms [13], fuzzy similarity measures [29] and rule learning [30].

Most of these techniques involve modifying the original input variables of fuzzy systems that can cause the loss of the original meaning of variables [17] and also reduce the model interpretability. Nevertheless, one effective way to deal with this problem is through the use of a special type of FLS, namely HFSs, that will reduce the number of rules while retaining the original meaning of variables [2], [21], [31].

### *B. Hierarchical Fuzzy Systems*

HFSs were initially proposed by Raju et al. [2], [31] as a solution to the rule explosion problem. They offer a distinct approach by organizing input variables into a set of low-dimensional FLSs, known as fuzzy logic subsystems [32]. This approach effectively addresses the complexity issue and enhances interpretability. Unlike traditional fuzzy systems, HFSs exhibit a linear rule increase instead of exponential growth [3]. In HFSs, assuming two input variables per low-dimensional fuzzy system and m fuzzy sets per variable, each low-dimensional fuzzy system comprises $m^2$ rules. Consequently, the total number of rules is represented by a linear function, $(n-1)m^2$, where n denotes the number of input variables [21]. Therefore, the number of rules in HFSs is always equal to or less than that of a comparable FLS. HFSs can be visualized as a cascade structure, where the output of each layer, referred to as the intermediate output, serves as an input for the subsequent layer [33], [34].

*1) How it works – HFS:* When managing a large-scale system, utilizing a hierarchical structure proves highly effective. This hierarchical arrangement of rules in fuzzy control allows for applying fuzzy controllers in relatively large systems [2]. The number of rules in the hierarchical structure can be reduced by dividing any level with three or more system variables into two levels, one containing only two system variables. This process can be repeated for all levels, resulting in the minimum number of rules when each level consists of only two system variables [35].

The advantage of HFS is its ability to reduce the number of fuzzy rules while maintaining a high level of system accuracy. This reduces computational complexity for fuzzy systems, particularly those with large-dimensional input variables. Both type-1 and type-2 fuzzy systems encounter the same challenges under the hierarchical structure. One challenge is selecting the input variables of systems during system modelling. The other challenge arises when determining which input variables enter the systems after establishing the hierarchical structure [36].

*2) Advantages of Hierarchical Fuzzy System:* In most cases, the underlying idea behind the construction of HFSs is to cope with the complexity of problems. Here are some other advantages of the implementation of the HFS.

(a) *Rule reduction:* A significant issue arises when the number of system inputs is large, leading to an exponential growth in the number of rules [37], [38]. This results in increased computational complexity in building a fuzzy system, known as the curse of dimensionality. However, this problem can be overcome by restructuring the fuzzy subsystems hierarchically, which leads to a linear increase in the number of rules and reduces computational complexity [39]. Brown and Harris also suggest using a hierarchical structure of fuzzy rule bases to achieve linear growth in the number of rules [40].
In summary, one of the main objectives of using HFSs is to minimize computational complexity, reduce the size of the rule base, and consequently decrease the need for large system memory and processing time [41].

(b) *Improving the Interpretability:* Recently, significant research has been done on the interpretability of HFSs. The primary goal of HFSs is to minimize computational complexity and the size of the rule base. By reducing model complexity, the interpretability of the system can be improved, as systems with fewer rules are easier to interpret [42]. Salgado and Cunha [43] also suggested that the small number of rules in each fuzzy sub-system can help avoid this problem without compromising global model accuracy. In some cases, input variables' linguistic terms and membership functions are assumed to be identical in each fuzzy sub-system. This assumption aims to maintain consistency in interpretation and simplify the model [23]. According to Benítez and Casillas, [44], HFSs have good interpretability due to several reasons: (i) the hierarchical structure results in a lower number of variables in each subsystem, (ii) the algorithm does not generate artificial linking variables, ensuring that all variables are interpretable because they belong to the system, and (iii) the rules are simpler because the number of variables per subsystem is lower. At the same time, accuracy is either maintained or improved. However, it is important to note that this statement is context-specific and cannot be generalized.

(c) *The trade-off between Accuracy and Interpretability:* According to a survey conducted by Shukla and Tripathi [45], the HFSs are considered one of the approaches that achieve a balance in the design of complex fuzzy systems, specifically in terms of rules, rule bases, membership functions, and fuzzy partitions. Delgado et al. [46] also demonstrated that HFSs can be adjusted to improve the model's performance and accuracy and ensure its interpretability. Using a hierarchical knowledge base, linguistic modelling aims to decompose a non-linear system that strikes a desired balance between interpretability and accuracy [26].
However, the decision on the model's level of interpretability and accuracy usually depends on the user's specific needs for a particular problem. It will influence the selection of the FLSs type for use [47].

(d) *Universal approximation:* Zeng and Keane [23] conducted a study on the approximation capabilities of HFSs. The analysis revealed that hierarchical fuzzy approximation, compared to standard fuzzy approximation, can greatly reduce the number of rules and parameters needed to achieve the desired level of accuracy. Wang [38] provided proof that a specific class of HFSs can serve as a universal approximator for any real continuous function on a compact set, and this finding was further supported by Joo and Lee [48].

## III. Comparison Between Fuzzy Logic Systems and Hierarchical Fuzzy Systems

This section attempts to elucidate this paper's principal aim: comprehensively comparing FLS and HFS methodologies. The Iris classification problem will be an example in this paper to thoroughly compare architectural aspects, including topologies, layers, and subsystems.

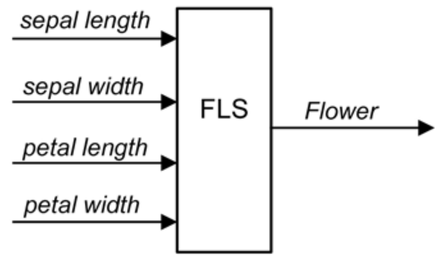


Fig. 1. FLS: Iris classification

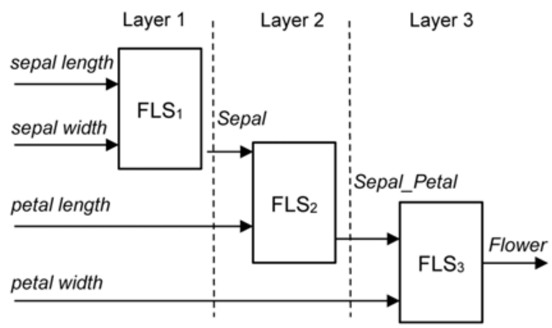


Fig. 2. Serial HFS: Iris classification

### A. Topologies

The Iris dataset, introduced by Fisher in 1936 [49], serves as an illustration to formally define the FLSs and HFSs. This dataset consists of four input variables: *sepal length (SL), sepal width (SW), petal length (PL)*, and *petal width (PW)*, as well as one output variable representing the flower type: *Sentosa, Versicolor, and Virginica*. Figure 1 depicts the topology of an FLS for iris classification, while Figures 2 and 3 show the topology of an HFS for the same purpose.

The concept of HFSs involves organizing the input variables into a set of low-dimensional FLSs, which are interconnected hierarchically. Different topologies like serial and parallel can create HFSs with the same input variables [44].

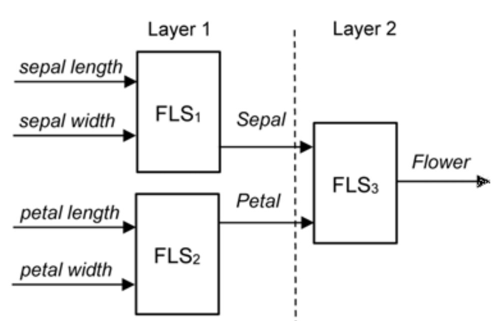


Fig. 3. Parallel HFS: Iris classificationn

### B. Layers

HFSs are generated by decomposing the input variables in FLSs into multiple low-dimensional FLSs, creating multiple layers within HFSs. In serial HFSs, each layer consists of one FLS, while in parallel HFSs, there can be more than one low-dimensional FLS per layer. This is illustrated in Figure 2 and Figure 3, respectively, where the two different HFS topologies using the same four input variables have different numbers of layers.

Campello and Amaral argue that for a topology with two input variables, the most parsimonious models are achieved by combining the output with another input variable into the second fuzzy system, repeating this process until all input variables are utilized [39]. On the other hand, Raju et al. suggest selecting the most influential input variables as system variables in the first layer, followed by the next most important variables in subsequent layers [2]. However, Wang's study indicates that there is no definitive conclusion regarding which inputs are more influential to the system output [38].

### C. Subsystem

HFSs are characterized by multiple subsystems contributing to the final solution's overall computation. Each subsystem is designed to have a limited number of inputs and outputs, a smaller rule base, and serves a specific purpose [50]. These subsystems are interconnected so that the output of one subsystem becomes the input for subsequent subsystems. For instance, in Figure 2, the subsystem FLS1 takes inputs of *sepal length* and *sepal width* and produces the output of *Sepal* classification. This subsystem is dedicated to the task of determining the classification of *Sepal*, and it operates with its own small rule base, which can be represented as follows:

- IF sepal length is small AND sepal width is small THEN Sepal is small
- IF sepal length is small AND sepal width is medium THEN Sepal is small
- IF sepal length is small AND sepal width is large THEN Sepal is medium
- IF sepal length is medium AND sepal width is small THEN Sepal is small
- IF sepal length is medium AND sepal width is medium THEN Sepal is medium
- IF sepal length is medium AND sepal width is large THEN Sepal is large
- IF sepal length is large AND sepal width is small THEN Sepal is medium
- IF sepal length is large AND sepal width is medium THEN Sepal is large
- IF sepal length is large AND sepal width is large THEN Sepal is large

## IV. Decomposition from Fuzzy Logic System to Hierarchical Fuzzy Systems

The HFS for iris classification is achieved by breaking down the input variables of the FLS, as depicted in Figure 1, into a set of subsystems, namely FLS1, FLS2 and FLS3, as illustrated in Figure 2 and Figure 3. Moreover, this HFS generates intermediate outputs such as *Sepal (S)*, *Sepal_Petal (S_P)*, and *Petal (P)*. HFSs can be seen as a functional decomposition of FLSs [3]. For example, the FLS and HFS for iris classification, shown in Figure 1 and Figure 2, respectively, can be described functionally as:

| FLS | HFS |
|---|---|
| Rules: | Rules: |
| IF *sepal length* is *small* AND *sepal width* is *small* AND *petal length* is *small* AND *petal width* is *small* THEN *Flower* is *Sentosa* | FLS1: IF *sepal length* is *small* AND *sepal width* is *small* THEN *Sepal* is *small*<br>FLS2: IF *petal length* is *small* AND *petal width* is *small* THEN *Petal* is *small*<br>FLS3: IF *Sepal* is *small* and *Petal* is *small* THEN *Flower* is *Sentosa* |

Fig. 4. Example rules in FLS and HFS

$$Flower = F(SL, SW, PL, PW) \Rightarrow Flower = f_3(f_1(SL, SW), f_2(PL, PW))$$

An FLS transitioning from a single layer, as depicted in Figure 1, to two layers, as shown in Figure 2, reduces the number of rules when considering a fully specified rule base. The most significant reduction in rules occurs when the HFS structure has two input variables for each low-dimensional FLS and incorporates $(n-1)$ layers [2], where n represents the total number of input variables, as illustrated in Figure 2. Assuming two input variables per low-dimensional FLS and m fuzzy sets for each input variable, including the intermediate output variables $y_1, \ldots, y_{(n-2)}$, the total number of rules ($R_{HFS}$ ) follows a linear function [21] in terms of the number of input variables n and can be formulated as:

$$R_{HFS} = (n-1)m^2. \tag{1}$$

Note that equation (1) applies only to HFS structures with two input variables per FLS subsystem in a serial structural form, as shown in Figure 2. In contrast, conventional FLSs experience an exponential increase in the number of rules with the number of input variables [51]. For a system with $n$ input variables and $m$ fuzzy sets for each input variable, the number of rules ($R_{FLS}$) (using the "AND" logical connective) can be represented as:

$$R_{FLS} = m^n \tag{2}$$

From these equations (1) and (2), it is clear that the total number of rules in the FLSs ($R_{FLS}$) is always greater than or equal to the equivalent HFSs ($R_{HFS}$). For example, Figure 1 and Figure 2 show an FLS and HFS with 4 input variables ($n = 4$) and, assuming that 3 fuzzy sets ($m = 3$) are defined for each input variable, the total number of rules for this FLS is $R_{FLS} = m^n = 3^4 = 81$ whereas for the HFS, the total number of rules is $R_{HFS} = (n-1)m^2 = (4-1)3^2 = 27$. This demonstrates that the HFS approach significantly reduces the number of rules compared to the FLS approach.

Figure 4 illustrates the rule structure comparison between the FLSs and HFSs for Iris classification. HFSs decompose the

TABLE I
RESULT OF INTERPRETABILITY COMPUTED USING HMEAN INDEX

| Iris classification | Index (*H*mean) |
|---|---|
| FLS | 0.194 |
| Parallel HFS | 0.493 |
| Serial HFS | 0.493 |

rules from FLSs into smaller rules within multiple subsystems, specifically FLS1, FLS2, and FLS3. This decomposition results in a simplified rule structure in HFSs, as each subsystem has fewer variables per rule than FLSs. As a result, the rules in HFSs are more straightforward and easier to comprehend, enhancing the human readability of the rule base [52].

### A. Interpretability

Razak et al., [20] proposed the *H* framework to evaluate the interpretability of the HFSs by combining interpretability assessments from each subsystem into a single overall metric. The *H* framework aims to provide a comprehensive measure of HFS interpretability.

$$Hmean = \sum_{j=1}^{q} \left( l_j \sum_{k=1}^{s_j} E_{jk}/s_j \right), \tag{3}$$

where $E_{jk}$ is the underlying (standard) FLS index associated with the subsystem $k$ at layer $j$, for example, the Fuzzy (F) index, $l_j$ is the weight associated with layer $j$ of the HFS, $s_j$ is the number of subsystems located in layer $j$, $s$ is the total number of subsystems and $q$ is the number of layers of the HFS.

The interpretability of the FLSs and HFSs, including both Parallel and Serial HFSs for the case of the Iris classification problem, was evaluated using the Hmean index. Table I shows the assessment results, indicating that both HFSs achieved a *higher* index than FLS. This suggests that HFSs are *more* interpretable than FLS, particularly in the context of the Iris classification example.

### B. Complexity

Razak et al. suggested a method for evaluating the complexity of HFSs that takes into account the complexity of its structure, which requires several subsystems, layers, and a dynamic topology [15], [53]. Also, the approach seems to be better with its combined structure and rule-based complexity. It can be computed as follows:

$$\mathrm{C}_{HFS} = \mathrm{C}_{RB} \bigoplus \mathrm{C}_{S} \tag{4}$$

where $\mathrm{C}_{RB}$ is rule-based complexity, $\mathrm{C}_S$ is structural complexity, and $\bigoplus$ indicates the generic aggregation operator such as min, max and mean. In this paper, we will use the mean in (4) to measure the complexity of HFSs. Further information for this (4) can be seen in [53], [54].

## V. DISCUSSION

The comparison between traditional FLS and the proposed HFS reveals critical insights into their respective advantages. HFS exhibits superior interpretability compared to FLS due to its hierarchical structure. This hierarchical organization allows complex systems to be decomposed into multiple levels of fuzzy rules, facilitating a more intuitive understanding of system behaviour. By breaking down the FLS into HFS, we can accurately represent the system's fundamental dynamics and decision-making processes. This enhanced interpretability is particularly valuable in practical applications where describing and comprehending system behaviour is essential.

Furthermore, the decomposition of FLS into HFS contributes to a reduction in system complexity. By partitioning the system into multiple levels, HFS simplifies the modelling process, with each level focusing on specific aspects of system behaviour. This simplification of fuzzy principles enhances overall system performance, offering a less complex representation that can be advantageous regarding computational resources and maintenance.

HFS emerges as a powerful tool for solving complex and hierarchical problems due to its enhanced interpretability and reduced complexity. Its hierarchical structure enables a detailed analysis of system behaviour, revealing the system's global and local aspects. Moreover, refining the output by considering local behaviours at the second level of HFS enhances the precision of system forecasts and decisions.

However, it is crucial to acknowledge the drawbacks of HFS. Introducing a hierarchical structure complicates the design and implementation process, requiring careful consideration of system characteristics and the establishment of appropriate norms at each level. The hierarchical strategy may also incur *higher* computational costs due to the added processing layers. Despite these challenges, the benefits of improved interpretability and reduced complexity position HFS as a promising approach for addressing complex and hierarchical problems.

In future research, further exploration into the practical implementation and optimization of HFS is warranted. Additionally, investigating strategies to mitigate the challenges associated with hierarchical design and computational costs would contribute to the broader applicability and effectiveness of HFS in various domains.

## VI. CONCLUSION

In conclusion, this study has rigorously compared Hierarchical Fuzzy Systems (HFS) with traditional Fuzzy Logic Systems (FLS), highlighting the significant advantages of HFS. HFS offers enhanced interpretability through its hierarchical structure and reduces system complexity by decomposing FLS into multiple levels. Despite challenges such as design complexity and potential computational costs, our findings underscore the potential of HFS as a practical methodology for addressing complex and hierarchical problems. Future research should prioritize optimizing hierarchical structure design, minimizing computational expenses, and exploring diverse applications of HFS across various domains. Overall, our study contributes valuable insights into the capabilities of HFS, paving the way for its broader adoption and increased effectiveness in real-world scenarios.

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